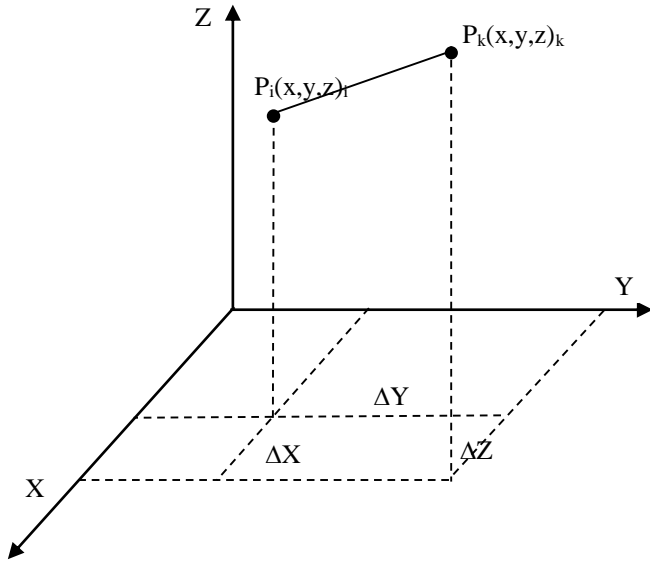


GPS AĞLARININ DENGELENMESİ

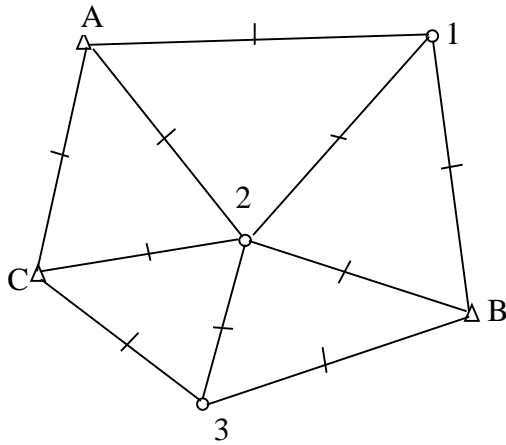


GPS Ölçüleri;

$\overline{P_i P_k}$ uzunluğuna ait $\Delta X_i, \Delta Y_i, \Delta Z_i$

İstenenler;

$X_i, Y_i, Z_i, X_k, Y_k, Z_k$



GPS ile ölçülenler;

$\overline{A1} \Rightarrow \Delta X_{A1}, \Delta Y_{A1}, \Delta Z_{A1}$

$\overline{1B} \Rightarrow \Delta X_{1B}, \Delta Y_{1B}, \Delta Z_{1B}$

$\overline{12} \Rightarrow \Delta X_{12}, \Delta Y_{12}, \Delta Z_{12}$

.....

$\overline{3C} \Rightarrow \Delta X_{3C}, \Delta Y_{3C}, \Delta Z_{3C}$

n ölçü sayısı = $3 \cdot 9$ baz = 27 ; u bilinmeyen sayısı = $3 \cdot 3 = 9$

$f = n - u = 27 - 9 = 18$ fazla ölçü var, dengeleme yapılır.

$$\Delta \hat{X}_{A1} = X_1 - X_A; \Delta \hat{Y}_{A1} = Y_1 - Y_A, \Delta \hat{Z}_{A1} = Z_1 - Z_A$$

$$\Delta \hat{X}_{1B} = X_B - X_1; \Delta \hat{Y}_{1B} = Y_B - Y_1, \Delta \hat{Z}_{1B} = Z_B - Z_1$$

.....

$$\Delta \hat{X}_i = \Delta X_i + V_{\Delta X_i}; \Delta \hat{Y}_i = \Delta Y_i + V_{\Delta Y_i}; \Delta \hat{Z}_i = \Delta Z_i + V_{\Delta Z_i}$$

$$\Delta X_{A1} + V_{\Delta X_{A1}} = X_1 - X_A; \Delta Y_{A1} + V_{\Delta Y_{A1}} = Y_1 - Y_A; \Delta Z_{A1} + V_{\Delta Z_{A1}} = Z_1 - Z_A$$

$$\Delta X_{1B} + V_{\Delta X_{1B}} = X_B - X_1; \Delta Y_{1B} + V_{\Delta Y_{1B}} = Y_B - Y_1; \Delta Z_{1B} + V_{\Delta Z_{1B}} = Z_B - Z_1$$

.....

$$\Delta X_{3C} + V_{\Delta X_{3C}} = X_C - X_3; \Delta Y_{3C} + V_{\Delta Y_{3C}} = Y_C - Y_3; \Delta Z_{3C} + V_{\Delta Z_{3C}} = Z_C - Z_3$$

Düzeltilme denklemleri;

$$V_{\Delta X_{A1}} = X_1 - X_A - \Delta X_{A1}; V_{\Delta Y_{A1}} = Y_1 - Y_A - \Delta Y_{A1}, V_{\Delta Z_{A1}} = Z_1 - Z_A - \Delta Z_{A1}$$

$$V_{\Delta X_{1B}} = X_B - X_1 - \Delta X_{1B}; V_{\Delta Y_{1B}} = Y_B - Y_1 - \Delta Y_{1B}, V_{\Delta Z_{1B}} = Z_B - Z_1 - \Delta Z_{1B}$$

.....

$$V_{\Delta X_{3C}} = X_C - X_3 - \Delta X_{3C}; V_{\Delta Y_{3C}} = Y_C - Y_3 - \Delta Y_{3C}, V_{\Delta Z_{3C}} = Z_C - Z_3 - \Delta Z_{3C}$$

X_i, Y_i, Z_i büyük değerler olduğu için yaklaşık değerler kullanılır.

$$X = X_0 + dx; Y = Y_0 + dy; Z = Z_0 + dz$$

$$X_1^0 = X_A + \Delta X_{A1}; Y_1^0 = Y_A + \Delta Y_{A1}; Z_1^0 = Z_A + \Delta Z_{A1}$$

$$X_2^0 = X_C + \Delta X_{C2}; Y_2^0 = Y_C + \Delta Y_{C2}; Z_2^0 = Z_C + \Delta Z_{C2}$$

$$X_3^0 = X_C + \Delta X_{3C}; Y_3^0 = Y_C + \Delta Y_{3C}; Z_3^0 = Z_C + \Delta Z_{3C}$$

$$\begin{array}{ll}
V_{\Delta X_{A1}} = dx_1 & -\ell_{\Delta X_{A1}}; -\ell_{\Delta X_{A1}} = X_1 - X_A - \Delta X_{A1} \\
V_{\Delta X_{A1}} = & dy_1 & -\ell_{\Delta Y_{A1}}; -\ell_{\Delta Y_{A1}} = Y_1 - Y_A - \Delta Y_{A1} \\
V_{\Delta X_{A1}} = & dz_1 & -\ell_{\Delta Z_{A1}}; -\ell_{\Delta Z_{A1}} = Z_1 - Z_A - \Delta Z_{A1} \\
\\
V_{\Delta X_{12}} = -dx_1 & + dx_2 & -\ell_{\Delta X_{12}}; -\ell_{\Delta X_{12}} = X_2 - X_1 - \Delta X_{12} \\
V_{\Delta X_{12}} = & -dy_1 & + dy_2 & -\ell_{\Delta Y_{12}}; -\ell_{\Delta Y_{12}} = Y_2 - Y_1 - \Delta Y_{12} \\
V_{\Delta X_{12}} = & -dz_1 & + dz_2 & -\ell_{\Delta Z_{12}}; -\ell_{\Delta Z_{12}} = Z_2 - Z_1 - \Delta Z_{12} \\
\\
..... \\
V_{\Delta X_{3C}} = & -dx_3 & -\ell_{\Delta X_{3C}}; -\ell_{\Delta X_{3C}} = X_C - X_3 - \Delta X_{3C} \\
V_{\Delta X_{3C}} = & -dy_3 & -\ell_{\Delta Y_{3C}}; -\ell_{\Delta Y_{3C}} = Y_C - Y_3 - \Delta Y_{3C} \\
V_{\Delta X_{3C}} = & -dz_3 & -\ell_{\Delta Z_{3C}}; -\ell_{\Delta Z_{3C}} = Z_C - Z_3 - \Delta Z_{3C}
\end{array}$$

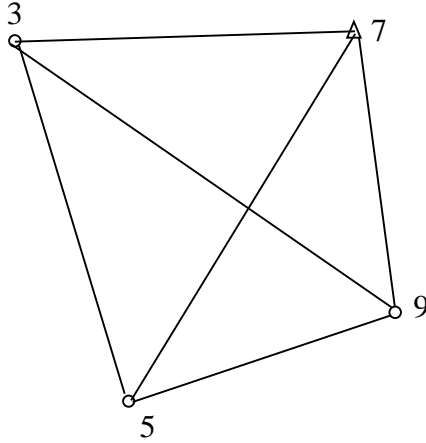
$$\begin{bmatrix} \mathbf{V}_{\Delta X_{A1}} \\ \mathbf{V}_{\Delta Y_{A1}} \\ \mathbf{V}_{\Delta Z_{A1}} \\ \mathbf{V}_{\Delta X_{12}} \\ \mathbf{V}_{\Delta Y_{12}} \\ \mathbf{V}_{\Delta Z_{12}} \\ \dots \\ \mathbf{V}_{\Delta X_{3C}} \\ \mathbf{V}_{\Delta Y_{3C}} \\ \mathbf{V}_{\Delta Z_{3C}} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -1 \end{bmatrix} \begin{bmatrix} d_{X_1} \\ d_{Y_1} \\ d_{Z_1} \\ d_{X_2} \\ d_{Y_2} \\ d_{Z_2} \\ \dots \\ d_{X_3} \\ d_{Y_3} \\ d_{Z_3} \end{bmatrix} - \begin{bmatrix} \ell_{\Delta X_{A1}} \\ \ell_{\Delta Y_{A1}} \\ \ell_{\Delta Z_{A1}} \\ \ell_{\Delta X_{12}} \\ \ell_{\Delta Y_{12}} \\ \ell_{\Delta Z_{12}} \\ \dots \\ \ell_{\Delta X_{3C}} \\ \ell_{\Delta Y_{3C}} \\ \ell_{\Delta Z_{3C}} \end{bmatrix} \Rightarrow \mathbf{V} = \mathbf{A} \mathbf{d} \mathbf{x} - \ell$$

$$\mathbf{K}_{\ell\ell} = \begin{bmatrix} \sigma_{\Delta X_{A1}}^2 & & & & & & & & & \\ & \sigma_{\Delta Y_{A1}}^2 & & & & & & & & \\ & & \sigma_{\Delta Z_{A1}}^2 & & & & & & & \\ & & & \sigma_{\Delta X_{12}}^2 & & & & & & \\ & & & & \sigma_{\Delta X_{12}}^2 & & & & & \\ & & & & & \sigma_{\Delta X_{12}}^2 & & & & \\ & & & & & & \sigma_{\Delta X_{3C}}^2 & & & \\ & & & & & & & \sigma_{\Delta X_{3C}}^2 & & \\ & & & & & & & & \sigma_{\Delta X_{3C}}^2 & \end{bmatrix} \quad \mathbf{P} = \frac{1}{m_0^2} \mathbf{K}_{\ell\ell}^{-1} = \mathbf{Q}_{\ell\ell}^{-1}$$

$$\mathbf{V}^T \mathbf{P} \mathbf{V} = \min. \text{ çözümünde } \mathbf{d} \mathbf{x} = \left(\mathbf{A}^T \mathbf{P} \mathbf{A} \right)^{-1} \mathbf{A}^T \mathbf{P} \ell ; \mathbf{X} = \mathbf{X}^0 + \mathbf{d} \mathbf{x}$$

$$\mathbf{Q}_{\mathbf{X}\mathbf{X}} = \left(\mathbf{A}^T \mathbf{P} \mathbf{A} \right)^{-1} m_0 = \pm \sqrt{\frac{[\mathbf{V}\mathbf{V}]}{n-u}} \quad \text{Birim ölçünün karesel ortalama hatası } m_x = \pm m_0 \sqrt{Q_{xx}} ; m_y = \pm m_0 \sqrt{Q_{yy}} ; m_z = \pm m_0 \sqrt{Q_{zz}}$$

Örnek :



GPS ile ölçülenler;

	ΔX	ΔY	ΔZ
$\overline{7-9}$	579.297	201.660	-898.280
$\overline{9-3}$	1337.369	-2947.357	2001.697
$\overline{3-5}$	2085.222	-853.834	-1575.283
$\overline{7-3}$	-758.051	-2745.673	1103.395
$\overline{5-9}$	-747.852	3801.192	-426.417

Kesin Değerler

NN	X	Y	Z
7	4213857.480	1025292.070	4664733.470

Ölçüler eşit duyarlıklıdır.

n ölçü sayısı = 3×5 baz = 15 ; u bilinmeyen sayısı = $3 \times 3 = 9$

$f = n - u = 15 - 9 = 6$ fazla ölçü var, dengeleme yapılır.

Noktaların yaklaşık koordinatları;

NN	X	Y	Z
9	4214436.777	1025493.730	4663835.190
3	4213099.408?	1025254.373	4665836.887
5	4215184.630	1021692.536	4664261.604

$$V_{\Delta X_{79}} = dx_9 \quad 0$$

$$V_{\Delta Y_{79}} = dy_9 \quad 0$$

$$V_{\Delta Z_{79}} = dz_9 \quad 0$$

$$V_{\Delta X_{93}} = -dx_9 \quad -dx_3 \quad 0$$

$$V_{\Delta Y_{93}} = -dy_9 \quad -dy_3 \quad 0$$

$$V_{\Delta Z_{93}} = -dz_9 \quad -dz_3 \quad 0$$

$$V_{\Delta X_{35}} = +dx_5 \quad -dx_3 \quad 0$$

$$V_{\Delta Y_{35}} = +dy_5 \quad -dy_3 \quad 0$$

$$V_{\Delta Z_{35}} = +dz_5 \quad -dz_3 \quad 0$$

$$V_{\Delta X_{73}} = +dx_3 \quad -21$$

$$V_{\Delta Y_{73}} = +dy_3 \quad -24$$

$$V_{\Delta Z_{75}} = +dz_3 + 22$$

$$V_{\Delta X_{59}} = dx_9 \quad -dx_5 \quad -1$$

$$V_{\Delta Y_{59}} = dy_9 \quad -dy_5 \quad -1$$

$$V_{\Delta Z_{59}} = dz_9 \quad -dz_5 \quad 3$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \end{bmatrix} \ell = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -21 \\ -24 \\ 22 \\ -1 \\ -1 \\ 3 \end{bmatrix}$$

$$V^T P V = \min. \text{ çözümünde } dx = (A^T P A)^{-1} A^T P \ell ; p = E$$

$$A^T A = \begin{bmatrix} 3 & 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 3 & 0 & 0 & -1 & 0 & 0 & -1 & 0 \\ 0 & 0 & 3 & 0 & 0 & -1 & 0 & 0 & -1 \\ -1 & 0 & 0 & 2 & 0 & 0 & -1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 2 & 0 & 0 & -1 & 0 \\ 0 & 0 & -1 & 0 & 0 & 2 & 0 & 0 & -1 \\ -1 & 0 & 0 & -1 & 0 & 0 & 3 & 0 & 0 \\ 0 & -1 & 0 & 0 & -1 & 0 & 0 & 3 & 0 \\ 0 & 0 & -1 & 0 & 0 & -1 & 0 & 0 & 3 \end{bmatrix}, A^T \ell = \begin{bmatrix} -1 \\ -1 \\ 3 \\ 1 \\ 1 \\ -3 \\ -21 \\ -24 \\ 22 \end{bmatrix}, dx = \begin{bmatrix} 8.11 \\ 9.25 \\ -8.75 \\ 10.00 \\ 11.50 \\ -9.50 \\ 13.11 \\ 15.00 \\ -13.50 \end{bmatrix}$$

$$X = X^0 + dx; Y = Y^0 + dy; Z = Z^0 + dz$$

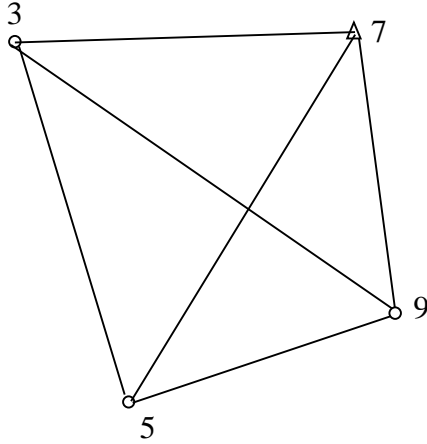
NN	X	Y	Z
9	4214436.785	1025493.739	4663835.181
3	4213055.421?	1025254.388	4665836.574
5	4215184.640	1021692.550	4664261.594

$$Q_{xx} = (A^T P A)^{-1} m_0 = \pm \sqrt{\frac{[VV]}{n-u}} \quad \text{Birim ölçünün karesel ortalama hatası}$$

$$m_x = \pm m_0 \sqrt{Q_{xx}}; m_y = \pm m_0 \sqrt{Q_{yy}}; m_z = \pm m_0 \sqrt{Q_{zz}}$$

GPS Ağlarının Ağırlıklı Dengelenmesi

Örnek :



GPS ile ölçülenler;

	ΔX	ΔY	ΔZ
$\overline{7-9}$	579.297	201.660	-898.280
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$\overline{3-5}$	2085.222	-853.834	-1575.283
$\overline{7-3}$	-758.051	-2745.673	1103.395
$\overline{5-9}$	-747.852	3801.192	-426.417

Kesin Değerler

<u>NN</u>	<u>X</u>	<u>Y</u>	<u>Z</u>
7	4213857.480	1025292.070	4664733.470

Ölçülerin Duyarlığı

n ölçü sayısı = 3×5 baz = 15 ; u bilinmeyen sayısı = $3 \times 3 = 9$

$f = n - u = 15 - 9 = 6$ fazla ölçü var, dengeleme yapılır.

Noktaların yaklaşık koordinatları;

NN	X	Y	Z
9	4214436.777	1025493.730	4663835.190
3	4213099.408?	1025254.373	4665836.887
5	4215184.630	1021692.536	4664261.604

Ölçülerin Duyarlılığı

	7-9	9-3	3-5	7-3	5-9
m_{ΔX}	±3.2 mm.	5.7	5.7	6.0	10.7
m_{ΔY}	±1.6 mm.	2.3	1.9	2.4	3.7
m_{ΔZ}	±4.7 mm.	4.5	4.4	5.5	8.7

$$r_{\Delta X \Delta Y} = 0.28; \quad r_{\Delta X \Delta Z} = 0.260; \quad r_{\Delta Y \Delta Z} = 0.37; \quad m_0^2 = 10 \text{ mm.}$$

$$V_{\Delta X_{79}} = dx_9 \quad 0$$

$$V_{\Delta Y_{79}} = dy_9 \quad 0$$

$$V_{\Delta Z_{79}} = dz_9 \quad 0$$

$$V_{\Delta X_{93}} = -dx_9 \quad -dx_3 \quad 0$$

$$V_{\Delta Y_{93}} = -dy_9 \quad -dy_3 \quad 0$$

$$V_{\Delta Z_{93}} = -dz_9 \quad -dz_3 \quad 0$$

$$V_{\Delta X_{35}} = +dx_5 \quad -dx_3 \quad 0$$

$$V_{\Delta Y_{35}} = +dy_5 \quad -dy_3 \quad 0$$

$$V_{\Delta Z_{35}} = +dz_5 \quad -dz_3 \quad 0$$

$$V_{\Delta X_{73}} = \quad \quad \quad + dx_3 \quad \quad -21$$

$$V_{\Delta Y_{73}} = \quad \quad \quad + dy_3 \quad -24$$

$$V_{\Delta Z_{75}} = \quad \quad \quad + dz_3 + 22$$

$$V_{\Delta X_{59}} = dx_9 \quad \quad -dx_5 \quad \quad -1$$

$$V_{\Delta Y_{59}} = \quad dy_9 \quad \quad -dy_5 \quad \quad -1$$

$$V_{\Delta Z_{59}} = \quad \quad dz_9 \quad \quad -dz_5 \quad \quad 3$$

$$A = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & -1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \end{bmatrix} \ell = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ -21 \\ -24 \\ 22 \\ -1 \\ -1 \\ 3 \end{bmatrix}$$

Ağırlıkların Hesabı;

$$K_{\ell\ell} = \begin{bmatrix} K_{79} & & & & \\ & K_{93} & & & \\ & & K_{35} & & \\ & & & K_{73} & \\ & & & & K_{59} \end{bmatrix}; K_{ij} = \begin{bmatrix} m_{\Delta Xij}^2 & m_{\Delta Xi\Delta Yj} & m_{\Delta Xi\Delta Zj} \\ m_{\Delta Xi\Delta Yj} & m_{\Delta Yij}^2 & m_{\Delta Yi\Delta Zj} \\ m_{\Delta Xi\Delta Zj} & m_{\Delta Yi\Delta Zj} & m_{\Delta Zij}^2 \end{bmatrix}$$

$$K_{79} = \begin{bmatrix} 3.2^2 & 0.28*3.2*1.6 & 0.6*3.2*4.7 \\ 0.28*3.2*1.6 & 1.6^2 & 0.37*1.6*4.7 \\ 0.6*3.2*4.7 & 0.37*1.6*4.7 & 4.7^2 \end{bmatrix} = \begin{bmatrix} 10.24 & 1.43 & 9.02 \\ 1.43 & 2.56 & 2.78 \\ 9.02 & 2.78 & 22.09 \end{bmatrix}$$

K_{93} , K_{35} , K_{73} , K_{59} bulunur.

$$V^T P V = \min. \text{ çözümünde } dx = (A^T P A)^{-1} A^T P \ell;$$

$$X = X^0 + dx; Y = Y^0 + dy; Z = Z^0 + dz$$

$$Q_{xx} = (A^T P A)^{-1} m_0 = \pm \sqrt{\frac{[VV]}{n-u}} \quad \text{Birim ölçünün karesel ortalama hatası}$$

$$m_x = \pm m_0 \sqrt{Q_{xx}}; m_y = \pm m_0 \sqrt{Q_{yy}}; m_z = \pm m_0 \sqrt{Q_{zz}}$$